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UNITED STATES PATENT APPLICATION  
OF

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FOR

SYSTEM AND METHOD FOR GENERATING A MODEL OF THE PATH OF A ROADWAY FROM AN  
IMAGE RECORDED BY A CAMERA

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1                   **INCORPORATION BY REFERENCE**

2                   U. S. Patent Application Serial No. 09/723,754, filed on November 26, 2000, in the names  
3                   of Gideon P. Stein, Ofer Mano And Amnon Shashua, and entitled "System And Method For  
4                   Estimating Ego-Motion Of A Moving Vehicle Using Successive Images Recorded Along The  
5                   Vehicle's Path Of Motion" (hereinafter referred to as "the Stein I patent application"), assigned to  
6                   the assignee of the present application, incorporated herein by reference.

7                   U. S. Patent Application Serial No. 09/723,755, filed on November 26, 2000, in the names  
8                   of Gideon P. Stein And Amnon Shashua, and entitled "System And Method For Generating A Model  
9                   Of The Path Of A Roadway From A Sequence Of Images Recorded By A Camera Mounted On A  
10                  Moving Vehicle" (hereinafter referred to as "the Stein II patent application") assigned to the assignee  
11                  of the present application, incorporated herein by reference.

12                   **FIELD OF THE INVENTION**

13                  The invention relates generally to the field of systems and methods for generating an estimate  
14                  as to the structure of a roadway from a vehicle and more specifically to systems and methods for  
15                  generating an estimate using an image recorded from the vehicle.

16                   **BACKGROUND OF THE INVENTION**

17                  Accurate estimation of the structure of a roadway ahead of a vehicle is an important  
18                  component in autonomous driving and computer vision-based driving assistance. Using computer  
19                  vision techniques to provide assistance while driving, instead of mechanical sensors, allows for the  
20                  use of the information that is recorded for use in estimating vehicle movement to also be used in  
21                  estimating ego-motion identifying lanes and the like, without the need for calibration between  
22                  sensors as would be necessary with mechanical sensors. This reduces cost and maintenance.

1        There are several problems in determining the structure of a roadway. Typically, roads have  
2        few feature points, if any. The most obvious features in a road, such as lane markings, are often  
3        difficult to detect and have a generally linear structure, whereas background image structures, such  
4        as those associated with other vehicles, buildings, trees, and the like, will typically have many  
5        feature points. This will make image- or optical-flow-based estimation difficult in practice. In  
6        addition, typically images that are recorded for roadway structure estimation will contain a large  
7        amount of "outlier" information that is either not useful in estimating roadway structure, or that may  
8        result in poor estimation. For example, in estimating of roadway structure, images of objects such  
9        as other vehicles will contribute false information for the road structure estimation. In addition,  
10        conditions that degrade image quality, such as raindrops and glare, will also make accurate road  
11        structure estimation difficult.

## SUMMARY OF THE INVENTION

13        The invention provides new and improved systems and methods for generating an estimate  
14        of the structure of a roadway using an image recorded from the vehicle.

15        In brief summary, the invention provides a road skeleton estimation system for generating  
16        an estimate as to a skeleton of at least a portion of a roadway ahead of a vehicle. The road skeleton  
17        estimation system includes an image receiver and a processor. The image receiver is configured to  
18        receive image information relating to at least one image recorded ahead of the vehicle. The  
19        processor is configured to process the image information received by the image receiver to generate  
20        an estimate of the skeleton of at least a portion of the roadway ahead of the vehicle.

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**BRIEF DESCRIPTION OF THE DRAWINGS**5  
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8  
This invention is pointed out with particularity in the appended claims. The above and further advantages of this invention may be better understood by referring to the following description taken in conjunction with the accompanying drawings, in which:9  
10  
11 FIG. 1 schematically depicts a vehicle moving on a roadway and including a roadway skeleton estimation constructed in accordance with the invention;12  
13  
14 FIG. 2 depicts a graph that schematically depicts a model of a roadway skeleton, useful in understanding one embodiment of the invention;15  
16  
17 FIG. 3 schematically depicts an image of a portion of a roadway, useful in understanding outputs performed by the roadway skeleton estimation system; and18  
19  
20 FIGS. 4 and 5 depict flow charts depicting operations performed by the roadway skeleton estimation system in estimating the skeleton of the roadway, FIG. 4 depicting operations in connection with a model in which the roadway is modeled as a circular arc and FIG. 5 depicting operations in connection with a model in which the roadway is modeled as a parabolic arc.21  
22  
23  
**DETAILED DESCRIPTION OF AN ILLUSTRATIVE EMBODIMENT**24  
25  
26  
27 FIG. 1 schematically depicts a vehicle 10 moving on a roadway 11 and including a roadway skeleton estimation system 12 constructed in accordance with the invention. The vehicle 10 may be any kind of vehicle 10 that may move on the roadway 11, including, but not limited to automobiles, trucks, buses and the like. The roadway skeleton estimation system 12 includes a camera 13 and a processor 14. The camera 13 is mounted on the vehicle 10 and is preferably pointed in a forward direction, that is, in the direction in which the vehicle would normally move, to record successive images as the vehicle moves over the roadway. Preferably as the camera 13 records each image, it will provide the image to the processor 14. The processor 14, in turn, will process

1 information that it obtains from the successive images, possibly along with other information, such  
2 as information from the vehicle's speedometer (not separately shown) to estimate a roadway skeleton  
3 representing a portion of the roadway 11 ahead of the vehicle 10. The processor 14 may also be  
4 mounted in or on the vehicle 11 and may form part thereof. The roadway skeleton estimates  
5 generated by the processor 14 may be used for a number of things, including, but not limited to  
6 autonomous driving by the vehicle, providing assistance in collision avoidance, and the like.  
7 Operations performed by the processor 14 in estimating the roadway skeleton will be described in  
8 connection with the flow chart depicted in FIG. 3.

9 Before proceeding further, it would be helpful to provide some background to the operations  
10 performed by the processor 14 in estimating the skeleton of the roadway 11. This background will  
11 be described in connection with FIG. 2. Generally, the roadway is modeled as a circular arc parallel  
12 to the XZ plane in three-dimensional space. The X (horizontal) and Y (vertical) axes of three-  
13 dimensional space correspond to the "x" and "y" axes of the image plane of the images recorded by  
14 the camera 14, and the Z axis is orthogonal to the image plane. Preferably, the image plane will be  
15 the plane of the image after the image has been rectified to provide that the Z axis is parallel to the  
16 plane of the roadway 11; the Stein I patent application describes a methodology for rectifying the  
17 images to provide that the image plane will have a suitable orientation.

18 As noted above, the roadway is modeled as a circular arc, and, with reference to FIG. 2, that  
19 FIG. circular arc in the XZ plane representing the roadway, which will be referred to as the  
20 roadway's skeleton, is identified by reference numeral S. Two lane markings 20L and 20R are also  
21 shown as dashed lines on opposite sides of the skeleton S. The distances R1 and R2 between the  
22 lane markings 20L and 20R and the skeleton S may be the same, or they may differ. The skeleton  
23 S of the roadway is modeled as an arc extending from the vehicle 10, or more specifically from the  
24 image plane of the camera 14, which is deemed to be at point  $P=(0,0)$ , the origin of the XZ plane.  
25 The center of the circular arc need not be along the X axis, and, indeed will generally be at an angle  
26  $\alpha$  thereto. The circular arc is parameterized by three components, namely

(i) the coordinates of the center ( $X_c$ ,  $Y_c$ ) of the circular arc (the coordinates of the center will also be referred to as coordinates (a,b));

(ii) the angle  $\alpha$  and the inverse of the radius  $1/R$ , and

(iii) for two locations Z1 and Z2 along the Z axis, values X1 and X2. In one embodiment, Z1 and Z2 are three meters and thirty meters, respectively.

For item (iii), value  $X1$  represents the horizontal distance between the  $Z$  axis, specifically the point with coordinates  $(0, Z1)$ , and the point on skeleton  $S$  with coordinates  $(X1, Z1)$ . A line  $L1$  is perpendicular to the line between point  $P=(0,0)$  and the center  $(Xc, Yc)$  of the circular arc comprising skeleton  $S$  intersecting the point  $P=(0,0)$ . The value  $X2$  represents the horizontal distance between the points on line  $L1$  and circular arc comprising skeleton  $S$  at coordinate  $Z2$ . In addition, it will be appreciated that the origin of the  $XY$  plane, point  $P=(0,0)$  also resides on the circular arc, so that the coordinates of three points on the circular arc comprising the skeleton  $S$ , along with the coordinates of the center of the circle that includes the circular arc, will be known. Parameterizing the arc  $S$  in this manner will have several advantages. First, if the locations  $Z1$  and  $Z2$  are relatively far apart, depending on the radius  $R$ , the values  $X1$  and  $X2$  may also be relatively far apart. Second, it would allow for use of a different road model, such as a parabolic arc, as will be described below.

Given a triplet  $(X_1, X_2, d)$ , where "d" is the pitch (that is, the angle of the camera relative to the Z axis) of the camera, and an image  $\Psi$ , it is desired to generate a warp of the image  $\Psi$  to a view in which the roadway is essentially a straight line. It is possible to warp the image onto the XZ plane, but instead the image is warped to the space  $R, \beta$ . Initially, given  $R$ ,  $a$  and  $b$ , it should be recognized that

$$R^2 = a^2 + b^2 \quad (1)$$

In addition

$$1 \quad X = (R + \Delta R) \cos(\beta + \beta_0) \quad (2)$$

2

and

3

$$Z = (R + \Delta R) \sin(\beta + \beta_0) \quad (3)$$

4

where  $\beta_0 = \tan^{-1}\left(\frac{a}{b}\right)$ .

5

In addition, since, for a point P(X,Y,Z) in three-dimensional space, the coordinates (x,y) of the projection of the point in the image are given by

$$6 \quad x = \frac{fX}{Z} \quad (4)$$

7

and

$$8 \quad y = \frac{fY}{Z} \quad (5)$$

9

10 where X and Z are determined as above and Y is a function of the camera height  $Y_h$ , the pitch "d"  
11 and the point's coordinate along the Z axis

12

$$Y = -Y_h + dZ \quad (6)$$

1 In equation (6),  $Y_h$  is a positive value and is subtracted from "dZ" since the surface of the roadway  
 2 11 is below the camera 14. It will be appreciated that, at the point  $Z=0$  directly below the camera,  
 3  $Y=Y_h$ , as required.

4 Since

5 
$$R^2 = (X_1 - a)^2 + (Z_1 - b)^2 \quad (7)$$

6 and

7 
$$R^2 = (X_2 - a)^2 + (Z_2 - b)^2 \quad (8)$$

8 and given the relation in equation (1), solving for "a" and "b"

9 
$$a = \frac{(X_2^2 + Z_2^2)Z_1 + (X_1^2 + Z_1^2)Z_2}{2(X_2Z_1 + X_2Z_2)} \quad (9)$$

10 and

11 
$$b = \frac{X_1^2 + Z_1^2 - 2aX_1}{2Z_1} \quad (10)$$

12 After values for "a" and "b," the "X" and "Z" components of the center of the circular arc S in the  
 13 XZ plane, the radius R can be determined using equation (1).

14 Using the values for "a" and "b" determined using equations (9) and (10), the value for the  
 15 radius R can be determined using equation (1). In addition, using equations (4) through (6) and the  
 16 coordinates (x,y) of each point in the image  $\Psi$ , the coordinates (X,Y,Z) of points in three-  
 17 dimensional space that are projected onto the image can be determined. Using the coordinates

1 (X,Y,Z), and the values for the radius R and  $\beta_0$ , the image can be warped from the (x,y) space to a  
 2  $\Delta R, \beta$  space using equations (2) and (3).

3 As noted above, the image  $\Psi$  in rectangular ((x,y)) coordinates can be warped to  $\Delta R, \beta$  space.  
 4 At this point, it is desired to determine the range and resolution of those parameters. In one  
 5 embodiment, it is desired to have the resolution on the order of one-tenth meter by one-tenth meter.  
 6 Accordingly, it is desired to determine  $\Delta\beta$  such that  $\Delta\beta R$  is on the order of a predetermined length,  
 7 which, in one embodiment, is one-tenth meter.

8 The range is determined as follows. As noted above, warping the image of the roadway to  
 9  $R, \beta$  space effectively provides a warped image  $\Psi'$  in which the roadway is straight, not curved. An  
 10 illustrative image is depicted in FIG. 3. As shown in FIG. 3, roadway 30 is divided into two regions,  
 11 including a near region 31 and a more distant region 32. In one embodiment, if the vertical  
 12 coordinates of the picture elements, or "pixels," that are subtended by the roadway in FIG. 3 extends  
 13 from -120 to -10, the vertical coordinates of the near region 31 extend from -120 to -20. In relating  
 14 the image in  $\Delta\beta, R$  space to the physical roadway in three-dimensional coordinates, it should be noted  
 15 that the bottom line of the warped image  $\Psi'$  maps to a line of width  $\Delta X$  at distance Z, where

$$16 \quad Z = \frac{fY}{y} \quad (11)$$

17 and, if, as shown in FIG. 3, the width of the image, with the horizontal coordinates extending from  
 18 -160 to 160 pixels, is 320

$$19 \quad \Delta X = \frac{\Delta xZ}{f} = \frac{\Delta x f Y}{y f} = \frac{\Delta x Y}{y} = \frac{320 Y_h}{-120} \quad (12)$$

20 The range of R, which is taken to be  $\Delta R$ , is  $3\Delta X$ .

21 In determining the range of  $\beta$ , it should be noted that  $Z_{\max}$ , the maximum value of Z for the  
 22 roadway, is

$$1 \quad Z_{\max} = \frac{fY}{-10} \quad (13).$$

2 Since

$$3 \quad \sin(\beta_{\max} + \beta_0) R = Z_{\max} \quad (14),$$

4 the value of  $\beta_{\max}$  is

$$5 \quad \beta_{\max} = -\beta_0 + \sin^{-1}\left(\frac{Z_{\max}}{R}\right) \quad (15).$$

6 With this background, operations performed by processor 14 will be described in connection  
 7 with the flow chart in FIG. 4. Generally, the operations proceed in two phases. In the first phase,  
 8 the processor 14 performs a rough alignment using a straight road model. In that operation, the areas  
 9 of the image  $\Psi'$  are detected that appear to belong to road direction indicators, such as lane markings,  
 10 tire marks, edges and so forth. In the second phase, the rough alignment generated during the first  
 11 series of steps is used, along with information in the image  $\Psi'$  including both regions 31 and 32 to  
 12 determine a higher-order model. In that operation, only features found in the distant region 32 that  
 13 are extensions of features found in the near region 31 are utilized, which will ensure that non-  
 14 roadway features, such as automobiles, that may be found in the distant region 32 will be ignored.

15 Accordingly, and with reference to FIG. 4, the processor 14 initially receives an image  $\Psi$   
 16 from the camera 14 (step 200). The image  $\Psi$  is a projection of points in rectangular three-  
 17 dimensional coordinates (X, Y, Z). After receiving the image  $\Psi$ , the processor 14 performs the first  
 18 series of steps in connection with the near region 31 (FIG. 3) of the roadway to generate a first-order  
 19 model of the roadway. Generally, for each of a predetermined number of selected values for  $X_1$ ,  
 20 keeping predetermined values of the other parameters  $X_2$  and radius R constant, the processor 14 will  
 21 generate a warped image  $\Psi'$ , and generates a value for a cost function for each warped image  $\Psi'$ . The

1 value of the cost function will represent a measure of the number of vertical features that are present  
2 in the warped image  $\Psi'$ . Since, for the warped image  $\Psi'$  for which the roadway appears to be  
3 straightest, the number of vertical features that are present in the warped image  $\Psi'$  will be largest,  
4 the processor 14 can select the value of the parameter  $X_1$  that was used in generating the warped  
5 image  $\Psi'$  associated with the largest cost function value as the appropriate value for parameter  $X_1$ .

6 Mores specifically, and with further reference to FIG. 4, after the processor 14 receives the  
7 image  $\Psi$ , it will select one, "i-th," of the selected values for parameter  $X_1$  (step 201) and using the  
8 selected value and the predetermined values for parameters  $X_2$  and  $R$ , generate a warped image  $\Psi'_i$ ,  
9 in which the image  $\Psi$  is warped to the  $R, \beta$  space (step 202). Thereafter, the processor 14 will  
10 generate the cost function, as follows. The processor 14 initially generates a derivative image  $d\Psi'$   
11 along the horizontal ("x") coordinate of the image  $\Psi'$ , the derivative representing the rate of change  
12 of image brightness, or intensity, as a function of the horizontal coordinate (step 203). Thereafter,  
13 to reduce the possibility that one extremely bright object might dominate, the processor 14 applies  
14 a non-linear function, such as a binary threshold or sigmoid function, to the derivative image  $d\Psi'$ ,  
15 thereby to generate a normalized derivative image  $n(d\Psi')$  (step 204). The processor 14, for each  
16 column of pixels in the normalized derivative image  $n(d\Psi')$  generated in step 204, generates a sum  
17 of the pixel values of the pixels in the respective column (step 205), generates, for each column, the  
18 square of the sum generated for the column (step 206) and forms a sum of the squares generated in  
19 step 206 as the value of the cost function for the value of the parameter  $X_1$  and the warped image  $\Psi'$   
20 generated therewith (step 207). In addition, the processor 14 determines, for each column, whether  
21 the value of the square exceeds a selected threshold (step 208).

22 Following step 208, the processor 14 will determine whether it has performed steps 202  
23 through 208 in connection with all of the selected values for parameter  $X_1$  (step 209). If the  
24 processor 14 makes a negative determination in connection with step 209, it will return to step 202  
25 to select another value for parameter  $X_1$  and performs steps 203 through 209 in connection therewith.

26 The processor 14 will perform steps 202 through 209 in connection with each of the selected  
27 values for parameter  $X_1$  to generate cost values therefor. When the processor 14 determines in step

1 209 that it has performed steps 202 through 209 in connection all of the selected values for parameter  
2  $X_1$ , it will sequence to step 209 to identify, among the cost function values generated in step 207,  
3 the maximum cost function value (step 210). In addition, the parameter 14 will determine the value  
4 of the parameter  $X_1$  and the warped image  $\Psi'$  associated with the maximum cost function value  
5 identified in step 209 (step 211). At this point the processor 14 will have completed the first phase,  
6 with the warped image  $\Psi'$  comprising the rough alignment.

7 After completing the first phase, the processor 14 begins the second phase. In the second  
8 phase, the processor 14 performs operations similar to those described above in the first phase,  
9 except that

10 (a) it performs the operations in connection with pixels not only in the near region 31, but  
11 also in the far region 32, and

12 (b) it generates the cost functions only in connection with columns that were determined in  
13 step 208 to exceed the threshold  
14 (step 212) to determine the values for the respective parameters.

15 The road skeleton S can also be modeled as a parabolic arc. In this case, it will be assumed  
16 that the major axis of the parabolic arc is along the X axis in three-dimensional space, which, as  
17 noted above, corresponds to the horizontal, or x, axis of the image  $\Psi$ . In that case, the road skeleton  
18 will conform to the equation

19 
$$X = aZ^2 + bZ + c \quad (16),$$

20 where coefficients "a," "b" and "c" are constants. The camera 14 is selected to be at the point  
21  $(X, Z) = (0, 0)$  on the skeleton S, in which case constant "c" is equal to zero. Lane markings are  
22 essentially horizontal translations from the skeleton, and therefore they can be modeled using  
23 equations of the same form as equation (16), and with the same values for coefficients "a" and "b" as  
24 that in the equation for the skeleton S, but different values for coefficient "c." It will be appreciated

1 that the value for coefficient "c" in the equations for the lane markings will indicate the horizontal  
2 displacement for the lane markings from the skeleton S for any point Z.

3 In the case of the parabolic arc model, the image  $\Psi$  can be warped to an image  $\Psi'$ , in which  
4 the skeleton is straight, as follows. If  $(x,y)$  are the coordinates of a point in image  $\Psi$  that is a  
5 projection of a point with coordinates  $(X,Y,Z)$  in three-dimensional space

$$6 \quad x = \frac{fX}{Z} \quad (17).$$

$$y = \frac{fY}{Z}$$

If a point in the image is a projection of a point the roadway,

$$8 \quad Y = -Y_h + dZ \quad (18).$$

9 where, as above (see equation (6)),  $Y_h$  is the height of the camera off the roadway and "d" is the pitch  
10 angle of the camera 14 relative to the horizontal axis Z. In equation (18),  $Y_h$  is a positive value, and  
11 is subtracted from "dZ" since the surface of the roadway 11 is below the camera 14. It will be  
12 appreciated that at the point  $Z=0$  directly below the camera,  $Y=Y_h$ , as required. If values for "f," the  
13 focal length of the camera 14,  $Y_h$  and d are known, it is possible to transform the coordinates  $(x,y)$   
14 of points in the image  $\Psi$  to the coordinates  $(X,Z)$  of points in three dimensional space, and vice  
15 versa. The values of the parameters f and  $Y_h$  are known and the value of parameter d will be  
16 determined in the course of determining values for coefficients "a" and "b" for the parabolic arc  
17 model equation (16). Also, given the values of coefficients "a" and "b" in equation (16), an overhead  
18 view  $(X,Z)$  of the skeleton of the roadway, that is, a view in the XZ plane, can be warped to a warped  
19 view  $(C,Z)$  in which the skeleton S has the equation  $C=0$ . In that case, roadway markings, which,

1 as noted above, are also modeled as parabolic arcs, will have equations  $C=c$ , where "c" is the value  
2 of the constant "c" in equation 16 for the respective roadway markings.

3 Accordingly, it is desired to determine the values of coefficients "a" and "b" and the pitch  
4 angle "d" that, when used to warp the overhead view  $(X,Z)$  of the skeleton  $S$  to the overhead view  
5  $(C,Z)$ , will result in a skeleton  $S$  that is a straight line. In that operation, a cost function is defined  
6 for the view  $(C,Z)$  whose value will be a maximum for the correct values of "a," "b" and "d." As  
7 with the methodology described above in connection with FIG. 4, searching for the maximum of the  
8 cost function's value is performed in two general phases, with a near region being used in the first  
9 phase, and a more extended region being used in the second phase. The value of the cost function  
10 is determined as follows. Given assumed values for "a," "b" and "d," the image  $\Psi$  is warped to an  
11 image  $\Psi'$ . The image  $\Psi$  is an image of the roadway (and possibly other objects and features) in  
12 which the skeleton of the roadway is, in an overhead  $(X,Z)$  view, a parabolic arc. On the other hand,  
13 image  $\Psi'$  is an image of the roadway (and possibly other objects and features) in which the skeleton  
14 of the roadway may, in an overhead  $(C,Z)$  view, be a straight line, depending on the values for "a,"  
15 "b" and "d" that were used in the warping. After the warped image  $\Psi'$  has been generated, the  
16 warped image is projected onto the warped image's horizontal "x" axis by summing the pixel values  
17 in the respective columns. The derivative of the sums across the warped image's horizontal "x" axis  
18 is then determined, and a sum of the absolute values of the derivative across the warped image's  
19 horizontal "x" axis is generated, with that sum being the value of the cost function. These operations  
20 are repeated for a number of assumed values for "a," "b" and "d" to generate a number of cost  
21 function values.

22 The value of cost function value will be larger if the projection onto the warped image's  
23 horizontal projection is sharper, which will occur if there are more vertical features in the warped  
24 image  $\Psi'$ , and it would be possible to select the warped image  $\Psi'$  for which the cost function value  
25 is greatest as the warped image  $\Psi'$  in which the skeleton  $S$  of the roadway appears as a straight line.  
26 If there were no objects in the image  $\Psi$ , and hence warped image  $\Psi'$ , other than the roadway 11, this  
27 would be correct.

1        However, if there is clutter, such as other objects, in the image  $\Psi$ , and hence warped image  
2  $\Psi'$ , with strong edges, the value of the cost function tends to get dominated by the clutter, which may  
3 result in an improper warped image  $\Psi'$  being selected. The two-phase methodology reduces the  
4 likelihood of this occurring. In the first phase, the image  $\Psi$  is warped to provide warped images  $\Psi'$   
5 using a straight road model, that is, with value of coefficient "a" set to zero and the values of "b" and  
6 "d" may be zero or non-zero. In each warped image  $\Psi'$ , areas that appear to belong to road direction  
7 indicators, such as lane markings and tire tracks, as opposed to lines associated with other objects,  
8 such as cars, are then identified. In addition, during the first phase, a preliminary value for  
9 coefficient "b" and a value for pitch "d" are determined.

10        Thereafter, in the second phase, image  $\Psi$  is again warped, to generate warped images  $\Psi''$ ,  
11 using selected values "a<sub>i</sub>" for coefficient "a" and the preliminary value for coefficient "b" and the  
12 value for pitch "d" that were determined during the first phase. Using areas vertically higher in the  
13 respective warped images, which represent areas in three-dimensional space (X,Y,Z) that are further  
14 from the vehicle 10, the cost function values are generated, and an assessment as to the warped  
15 image  $\Psi''$ , for which the skeleton S most approximates a straight line. The value "a<sub>i</sub>" that was used  
16 in generating the warped image  $\Psi''$ , for which the skeleton S most approximates a straight line is  
17 selected as the value for coefficient "a" for the model represented by equation (16). The value pitch  
18 "d" identified in the first phase is selected as the pitch "d." The value of coefficient "b" value for the  
19 model represented by equation (16) corresponds to the value  $\hat{b}$  which, as the coefficient of the linear  
20 term in the parabolic model for which the value "a<sub>i</sub>" is identified as the coefficient "a" in the  
21 parabolic model, most approximates the straight line model, using the preliminary value for "b," that  
22 was developed during the first phase.

23        More specifically, and with reference to the flow chart in FIG. 5, after the processor 14  
24 receives an image  $\Psi$  (step 250), it will, in the first phase, select initial guess for the values for pitch  
25 "d" and coefficient "b" (step 251) and, using those values, warp the image  $\Psi$  (step 252) thereby to  
26 generate a warped image  $\Psi'$ . The processor 14 then, for each column of pixels in the warped image  
27  $\Psi'$ , sums the pixel values for the pixels in the near region 31 in the column (step 253), generates

1 values representing the derivative in the horizontal direction (step 254) and generates values that  
2 correspond to the absolute value of the derivative (step 255). It will be appreciated that portions of  
3 the warped image  $\Psi'$  that contain, for example, edges or lane markings will produce peaks in the  
4 absolute values generated in step 255, since the edges and lane markings will correspond to fast  
5 transitions from dark-to-light or light- to-dark regions of the images. The processor 14 then  
6 thresholds the absolute values generated in step 255 (step 256), that is, for any absolute value that  
7 is less than a predetermined threshold value, the processor 14 sets the absolute value to zero. For  
8 the absolute values that are above the threshold, the processor 14 then finds local maxima (step 257).  
9 The local maxima, which will be identified as peaks " $p_i$ " typically correspond to lane markings in  
10 the warped image  $\Psi'$ , although some peaks  $p_i$  may be caused by a strong edge that is not part of the  
roadway, such as an automobile a short distance ahead of the vehicle 10.

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After identifying the peaks  $p_i$ , the processor 14 searches for values for the coefficient "b" and pitch "d" using a search methodology for each of the peaks  $p_i$  (step 258). In that operation, in each of a series of iterations "j" ( $j=1, \dots, J$ ), the processor 14 selects a value  $d_j$  for pitch d. In each iteration "j," the processor 14, in each of a series of iterations "k" ( $k=1, \dots$ ), selects a value  $b_k$  and performs steps 252 through 257 in connection therewith to find values  $b_k^i$  for which the respective peaks  $p_i$  are maximum. After completing the iterations "j" the set of points  $(d_j, b_k^i)$  in a (d,b) plane that are associated with each peak  $p_i$  form a line  $l_i$ . The lines  $l_i$  that are associated with features along the roadway, such as lane markings and tire tracks, will intersect at or near a point  $(d_x, b_x)$  in the (d,b) plane, and the processor 14 will identify that point and determine the appropriate values for coefficient "b" and pitch "d" as  $b_x$  and  $d_x$ , respectively. If there are any objects that are not associated with features of the roadway 11, such as automobiles, in the near region 31 that are associated with peaks  $p_i$ , lines  $l_i$  that are associated therewith will generally not be near point  $(d_x, b_x)$  and will be ignored. In addition, it will be appreciated that, since the lines  $l_i$  that are at or near point  $(d_x, b_x)$  are associated with features along the roadway, and the processor 14 can readily determine which columns in the warped image  $\Psi'$  contain those features. This, along with the values of coefficient "b" and pitch "d" will be used in the second phase.

1        In the second phase, the processor 14 determines a value for coefficient "a" using regions of  
2        the image including both the near region 31 and the far region 32, generally emphasizing columns  
3        that are associated with or near the subset of peaks  $p_i$  that it identified as being associated with  
4        features of the roadway. In the second phase, the processor 14 initially selects a plurality of values  
5         $a_i$  (step 260) and, for each value  $a_i$ , generates an adjusted value  $\hat{b}_i$  so that the line  $X = b_x Z + c$   
6        (where  $b_x$  corresponds to  $b_x$  determined in the first phase) best approximates the curve  
7         $X = a_i Z^2 + \hat{b}_i Z + c$  with the value of "c" being set to zero (reference equation 16) over a range  
8        of "Z" as determined in the first phase (step 261). Thereafter, the processor 14 selects a value of " $a_i$ "  
9        (step 262) and warps the original image  $\Psi$  to generate a warped image  $\Psi'$ ; using the values " $a_i$ " and  
10        " $b_x$ " as coefficients "a" and " $\hat{b}_i$ " and value  $d_x$  as the pitch "d" (step 263). The processor 14 then  
11        projects a portion of the warped image  $\Psi'$ , specifically the portion in the far region 32, onto the  
12        warped image's horizontal axis by summing the pixel values of the pixels in each column in that  
13        region (step 264) and then generates the derivative of the sums along the horizontal axis (step 265).  
14        For columns that are associated with peaks  $p_i$  that the processor 14 had determined in the first phase  
15        were associated with features of the roadway 11, such as lane markings and tire tracks, the processor  
16        14 will generate the absolute value of the derivative (step 266) and generate a value corresponding  
17        to the sum of the absolute values (step 267). It will be appreciated that, by performing steps 266 and  
18        267 in connection only with columns that were determined in the first phase to be associated with  
19        features of the roadway 11, the processor 14 will minimize contributions due to features that are not  
20        associated with the roadway, such as automobiles, which may otherwise unduly influence the result.

21        After performing steps 262 through 267 for the value " $a_i$ " that was selected in step 262, the  
22        processor 14 will determine whether it has selected all of the values " $a_i$ " that were selected in step  
23        260 (step 268), and, if not, return to step 262 to select another value " $a_i$ " and perform steps 263  
24        through 267 in connection therewith. The processor will perform steps 262 through 267 through a  
25        plurality of iterations until it has generated the sum of the absolute value of the derivative for all of  
26        the values " $a_i$ " that were selected in step 260. When the processor 14 determines in step 268 that it

1 has selected all of the values " $a_i$ " that were selected in 260, it will have generated a sum of the  
2 absolute value of the derivative for all of the values " $a_i$ ," in which case it will sequence to step 269.  
3 In step 269, the processor identifies the value " $a_i$ " for which the sum is the largest (step 269). The  
4 correct parameters for the skeleton S are, as the value of coefficient "a," the value  $a_i$  that was selected  
5 in step 269, as the value of coefficient "b," the value of  $\hat{b}$  that was generated for that value " $a_i$ " in  
6 step 261, and, as the pitch "d," the value  $d_x$  generated in the first phase.

7 The invention provides a number of advantages. In particular, the invention provides a  
8 system for estimating the skeleton S of a roadway 11 for some distance ahead of a vehicle. Two  
9 specific methodologies are described, one methodology using a model in which the roadway is  
10 modeled as a circular arc, and the other methodology using a model in which the roadway is  
11 modeled as a parabolic arc, in both methodologies requiring only one image, although it will be  
12 appreciated that the system can make use of a combination of these methodologies, and/or other  
13 methodologies. Determining the skeleton of a roadway for some distance ahead of a vehicle can be  
14 useful in connection with autonomous or assisted driving of the vehicle.

15 It will be appreciated that a system in accordance with the invention can be constructed in  
16 whole or in part from special purpose hardware or a general purpose computer system, or any  
17 combination thereof, any portion of which may be controlled by a suitable program. Any program  
18 may in whole or in part comprise part of or be stored on the system in a conventional manner, or it  
19 may in whole or in part be provided in to the system over a network or other mechanism for  
20 transferring information in a conventional manner. In addition, it will be appreciated that the system  
21 may be operated and/or otherwise controlled by means of information provided by an operator using  
22 operator input elements (not shown) which may be connected directly to the system or which may  
23 transfer the information to the system over a network or other mechanism for transferring  
24 information in a conventional manner.

25 The foregoing description has been limited to a specific embodiment of this invention. It will  
26 be apparent, however, that various variations and modifications may be made to the invention, with  
27 the attainment of some or all of the advantages of the invention. It is the object of the appended

1 claims to cover these and such other variations and modifications as come within the true spirit and  
2 scope of the invention.

3 What is claimed as new and desired to be secured by Letters Patent of the United States is: